THIS ISN’T YOUR FATHER’S PLANE GEOMETRY

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Paper folding activities provide students with a challenging and interesting path for discovery by allowing students to physically manipulate geometric figures. Introductory work associated with developing basic geometric concepts including geometric constructions that are sometimes difficult for the student new to geometry can be made simpler by using paper folding activities. The following paper folding activities are taken from *Discovering Geometry An Inductive Approach*, Second Edition, by Michael Serra published by Key Curriculum Press in 1997.

**Construct an Angle Bisector**

**Step 1**
On a patty paper, draw a large acute angle. Label it $PQR$.

**Step 2**
Fold your patty paper over so that $\overline{QP}$ and $\overline{QR}$ coincide. Crease your patty paper along the fold.

**Step 3**
Unfold your patty paper. Draw a ray with endpoint $Q$ along the crease. Is the ray the angle bisector of $\angle PQR$? (Why? Because the fold formed two angles that coincided.)

Clearly, an angle has a bisector. Does an angle have only one bisector? Can you find another ray that also bisects the angle? What else is true about an angle bisector? How would you describe the relationship between the points on the angle bisector and the sides of the angle being bisected?

If a point is on the bisector of an angle, then it is —?— from the sides of the angle.

*Discovering Geometry An Inductive Approach*
Michael Serra
1997 Key Curriculum Press
Construct a Perpendicular Bisector

Step 1
Draw a segment on a patty paper. Label it \( \overline{PQ} \).

Step 2
Fold your patty paper over so that endpoints \( P \) and \( Q \) coincide (and exactly on top of each other). Crease your paper along the fold.

Step 3
Unfold your paper. Draw a line in the crease. Does the line appear to be the perpendicular bisector of \( \overline{PQ} \)? Check with your ruler and protractor to verify that the line in the crease is indeed the perpendicular bisector of \( \overline{PQ} \).

How would you describe the relationship of the points on the perpendicular bisector with the endpoints of the bisected segment? Let's perform one more step in our investigation.

Step 4
Place three points on your perpendicular bisector. Label them \( A \), \( B \), and \( C \). With your compass, compare the distances \( PA \) and \( QA \). Compare the distances \( PB \) and \( QB \). Compare the distances \( PC \) and \( QC \).

What do you notice about the two distances from each point on the perpendicular bisector to the endpoints of the segment? Compare your results with the results of others near you. You should now be ready to state your first conjecture.

If a point is on the perpendicular bisector of a segment, then it is —?— from the endpoints

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Construct a Perpendicular to a Line Through a Point not on the Line

On a piece of patty paper, perform the steps below.

Step 1  Draw and label line $AB$ and a point $P$ not on $AB$.

Step 2  Fold the line on top of itself and slide until . . . well, you figure out the rest.

Construct a Line Parallel to a Given Line through a Point not on the Given Line

Step 1  Line and point.

Step 2  Fold perpendicular.

Step 3  Fold perpendicular to the perpendicular.

Step 4  TA DA!

Discovering Geometry An Inductive Approach
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FOLDING PAPER CIRCLES

Prerequisites
Students need to be familiar with the following conjectures:

- The measure of an angle inscribed in a circle is half the measure of its intercepted arc.
- In a circle, congruent chords are equidistant from the center and intercept congruent arcs.
- The midsegment of a triangle is parallel to the third side and one half its length.

Materials needed

Large paper circles (students can make these using blank paper, a compass, and a pair of scissors).

Taken from *Discovering Geometry Teacher's Resource Book*
1997 by Key Curriculum Press
Activity contributed by Sharon Grand
Folding Paper Circles

Sharon Grand

Use the steps below to fold a circle.

1. Cut a large circle from a piece of unlined paper. Mark the center.

2. Fold a point of the circle to the center. Call the crease segment $AB$. How far from the center is the chord? Identify the major and minor arcs for chord $AB$.

3. Use $B$ as an endpoint of a second crease. Fold a second point of the circle to the center (segment $BC$). How far from the center is the chord? Identify the major and minor arcs for chord $BC$. What is true about the lengths of the chords $AB$ and $BC$?

4. Fold segment $AC$. What kind of triangle is $ABC$? How do you know? Compare the three minor arcs. What are their measures? Compare the measures of the angles of the triangles to the measures of the arcs.

5. Locate the midpoint of segment $AB$ (point $D$). Fold point $C$ to point $D$. Crease to form segment $EF$. What kind of quadrilateral is $ABFE$? Explain how you know this.

6. Fold $B$ across to point $E$ along segment $DF$. Identify $ADFE$. Explain how you know this.

7. Fold point $A$ across to point $F$ along segment $DE$. What type of triangle did you form? How do you know? How does it compare to the first triangle? Unfold and compare the areas of the two triangles.

8. Open back to triangle $ABC$. Form a three-dimensional solid by folding along $DE$, $EF$, and $DF$. Discuss the three-dimensional shape that you formed.

9. Fold point $A$ to the mid-point of $DE$, point $B$ to the mid-point of $DF$, and point $C$ to the mid-point of $EF$. Slip the smallest triangle flaps (points $A$, $B$, and $C$) inside each other to make a truncated tetrahedron.

10. Unfold to the original circle. Discuss the geometric concepts/figures shown within the folded lines. Decorate your final product colorfully to show a particular geometric property.

Taken from *Discovering Geometry Teacher's Resource Book*  
1997 by Key Curriculum Press  
Activity contributed by Sharon Grand
Transformations

Geometry is not only the study of figures; it is also the study of the movement of figures. If you move all the points of a geometric figure according to set rules, you can create a new geometric figure. Such a motion establishes a correspondence between the points of the original figure and the points of the new figure. This new figure is called the image. If each point of a plane figure can be paired with exactly one point of its image on the plane, and if each point of the image can be paired with exactly one point of the original figure, then the correspondence is called a transformation.

A transformation that preserves size and shape is called a rigid transformation, or an isometry (from the Greek for “same measure”). The image is always congruent to the original in an isometry.

Three types of isometries are translation, rotation, and reflection. More familiar words for these motions are slide, turn, and flip. You have been working informally with translations, rotations, and reflections in your patty-paper investigations and in some motion exercises on the coordinate plane.

A translation is the simplest type of isometry. If you copy a figure onto a piece of paper, then slide the paper along a straight path without turning it, your slide motion models a translation isometry. In a slide, points in the original figure move an identical distance along parallel paths to the image. That is, all the points in the original figure are equidistant from their images. A distance and a direction together define a translation. You can use an arrow, representing a translation vector, to show distance and direction. The length of the translation vector from starting point to tip represents the distance, and the direction in which the arrow is pointing represents the direction of the translation.

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A translation can also be represented on a coordinate grid by using an ordered-pair rule. For example, the ordered pair rule \((x, y) \rightarrow (x+5, y+3)\) moves each point of the original figure five units to the right and three units up. We sometimes abbreviate the rule by simply writing the ordered pair \((5, 3)\), which we read as “translation: 
\((5, 3)\).” The figure below left shows a translation: \((5, 3)\). What translation is shown in the figure below right?

A prime mark (‘) is often used with the label of an image point. The image of point \(A\) under a transformation of any type is often called point \(A’\) (read “\(A\) prime”).

If you place a piece of patty paper over a figure and trace it (making an image of the original), then place a dot on the patty paper, put your pencil point on the dot, and rotate the patty paper about the point, your turn motion models a rotation isometry.

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A reflection is a third type of isometry. If you draw a figure onto a piece of paper and fold the edge of a mirror onto your paper so that the mirror is perpendicular to the paper, and look at the figure in the mirror, you will see the figure reflected. A reflection produces a mirror image. A line of reflection (also called a mirror line) defines a reflection. The line of reflection is the perpendicular bisector of every segment joining a point in the figure with the image of the point. If a point in the figure is on the line of reflection, then the point is its own image. Each point in a figure is reflected or flipped over the reflection line.

A reflection can be represented on a coordinate grid by using an ordered pair rule. For example, the ordered pair rule \((x, y) \rightarrow (x, -y)\) reflects each point of the original figure to the opposite side of the \(x\)-axis.

You can also model a reflection with patty paper. Sketch a figure onto a sheet of patty paper. Draw a line of reflection onto your patty paper. Fold the patty paper along the line of reflection. Trace the figure onto the folded portion of your patty paper. Unfold.

Combining a translation with a reflection gives a special two-step isometry called a glide reflection. A sequence of footsteps is a common example of a glide reflection. In Lesson 8.2, you’ll experiment with this and other combined transformations.

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