Research Interests

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1. Introduction and Background

Suppose S is a parametrized surface in complex projective 3-space $\mathbf{P}^3$ given as the image of $\phi : \mathbf{P}^1 \times \mathbf{P}^1 \to \mathbf{P}^3$. The map ϕ is known as a parametrization of the surface. The implicitization problem is to compute an implicit equation $F = 0$ of S using the parametrization ϕ. Parametric surfaces are widely used in Computer Aided Design projects since it is easy to describe the points of the surface by means of the parameter value. However, to describe the set of points which are common to two different parametrically presented surfaces is a difficult problem using the parametric descriptions. If the surfaces are described by means of external, i.e. implicit equations, then to find the set of common points of two surfaces reduces to the problem of finding the common solutions of two explicitly given polynomial equations. This is a problem which can be handled relatively easily. Thus there is a need for being able to go back and forth between a parametric and an implicit description of a surface. This is, in essence, the implicitization problem. Describe algorithms to produce an implicit equation of a surface for which one knows a parametric description.

Assuming we are given the parametrization ϕ, there are three methods to find the implicit equation: Gröbner basis, resultants, and syzygies. Only the case of syzygies are considered here. The syzygy technique was developed recently. The first introduction of syzygy-like techniques in the implicitization problem was the introduction by Sederberg and Chen [10] of the use of moving lines to produce implicit equations for curves. Shortly after that, Cox, Goldman and Zhang extended these ideas to show the validity of implicitization by moving quadrics for rational surfaces with no base points. No base points means that the parametric equation is defined for all values of the projective parameter. Later on, Cox [4] provided an algorithm for finding an implicit equation of a rational surface without base points via the theory of syzygies, which is a commonly used technique in Algebraic Geometry. Recently, Cox, Busé, and D'Andrea [3] produced an algorithm for finding an implicit equation of a rational surface given by a parameterization $\phi : \mathbf{P}^2 \to \mathbf{P}^3$ which has base points. The main result of Adkins-Hoffman-Wang [1] provides an algorithm for finding an implicit equation for a parametrized surface of the form $\phi : \mathbf{P}^1 \times \mathbf{P}^1 \to \mathbf{P}^3$ in which special kinds of base points are present. It is an extension of the algorithm of Cox [4], which did not allow base points. For this to work it was essential to have an extension of the basic concepts and results of regularity to the new situation of a bigraded polynomial algebra as opposed to simple graded algebra, in the classical setting. This was developed in the paper by Hoffman-Wang [7]. It is assumed that the base point ideals are local complete intersections, and for this one needed a theorem concerning the structure of the syzygies vanishing at these base points. In [6], Hoffman-Wang provided an analog of the theorem of Cox and Schenck [5] concerning the structure of the syzygies vanishing at certain base points is proven, and this is the tool needed.

2. Current and Future Research Interests and Projects

In the process of working on implicitization problems, many interesting questions arise. The following are the projects I am interested in:

Project 1: Although we proved the implitization algorithm works for the case of curvilinear base points, can we remove the curvilinear condition from the base points condition? Can we have a result of vanishing syzygies and local complete intersection as the result from [5]? Currently, I am investigating the theorem without curvilinear condition, especially a family of certain monomial parametrization with non-curvilinear base points. Bruening-Wang [2] have been found the implicit equation of this type of parametrization, and further study on the syzygy method is in progress.

Project 2: In the past year, there have been significant extensions of the concept of regularity. In a special case, this is due to Hoffman-Wang [7]. Recently Maclagan and Smith have developed a theory of regularity for toric varieties. Many, but not all, of the theorems in the classical setting have been extended to this new situation. One important open question is the relation of this new regularity concept to the structure of free resolutions for modules over certain kinds of multigraded algebras. Hoffman-Wang have introduced an extension of the regularity concept, called *strong regularity.* In the special case of bigraded polynomial rings $R = k[x_0, ..., x_m; y_0, ..., y_n]$ they have proved that strong regularity for a bigraded R-module M is equivalent to the existence of a certain type of free bigraded resolution for M. The theory of Maclagan-Smith [9] gives a weaker result, but in a more general context - for the multigraded rings arising from toric varieties. Sidman-Van Tuyl-Wang [8] extended some of techniques by Hoffman-Wang in [7] to obtain finite bounds on the multidegrees of a minimal free resolution. Hoffman-Wang are investigating the concept of strong regularity for these more general multigraded rings and investigate the relationship to free resolutions of modules over these rings. These are important in the applications to algorithms for solving polynomial systems.

References

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