Solving Quadratic Equations

**Quadratic Equation:** An equation that can be put in the form $ax^2 + bx + c = 0$. This is called the *standard form* of a quadratic equation.

Three methods to solve:

1. **Solve by factoring**
   a. Write in standard form
   b. Factor
   c. Set factors = 0
   d. Solve results.
   e. Check.

   **Example:** $x^2 + 8x = -15$
   - Write in standard form: $x^2 + 8x + 15 = 0$
   - Factor: $(x + 5)(x + 3) = 0$
   - Set factors = 0: $x + 5 = 0 \quad x + 3 = 0$
   - Solve results: $x = -5 \quad x = -3$
   - Check. Do it!!

2. **Solve by Completing the square**

   **Example:** $2x^2 - 12x = 8x - 38$
   - If already in $(something)^2 = number$, go to step f. Otherwise write with variables on left, number on right.
   - $(2x^2 - 20x = -38)$
   - If the coefficient of $x^2$ is one, go to step c. Otherwise divide both sides of the equation by the coefficient of $x^2$
   - Divide the $x$ coefficient by 2 (and get 5 in our case), and square it. $5^2 = 25$. Add the result to both sides.
   - Divide by 2: $x^2 - 10x = -19$
   - Add 25 to both sides: $x^2 - 10x + 25 = -19 + 25$
   - Factor the left side (which will always be a perfect square)
   - Factor the left side: $(x - 5)^2 = 6$
   - Apply the square root property to the equation.
   - $\sqrt{(x - 5)^2} = \pm\sqrt{6}$
   - Simplify and solve
   - $x - 5 = \pm\sqrt{6}$
   - So $x = 5 \pm \sqrt{6}$
   - Or you can write
   - $x = 5 + \sqrt{6}$ or $x = 5 - \sqrt{6}$
   - CHECK!
3. Solve using the **quadratic formula**: \( 2x^2 + 14x = 22x - 3 \)

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<tbody>
<tr>
<td>a. Write in standard form ( ax^2 + bx + c = 0 )</td>
<td>( 2x^2 - 8x + 3 = 0 )</td>
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<tr>
<td>b. Divide out common numerical factors</td>
<td><em>no common factors this time</em></td>
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| c. Matching to the standard form pattern, substitute \( a, b, \) and \( c \) into the quadratic equation. \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) | \( a = 2 \)
| c. = 3 | \( b = -8 \)
| c. = 3 | \( c = 3 \)
| d. Simplify | \( x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)} \) |
| The answer is | \( x = 2 \pm \frac{\sqrt{10}}{2} \) or you can write them as separate answers as in 2f |

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<td>e. Check!!</td>
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4. Here is the example used in the Completing the Square directions, redone using the **quadratic formula**: \( 2x^2 - 12x = 8x - 38 \)

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<td>( 2x^2 - 20x + 38 = 0 )</td>
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<tr>
<td>b. Divide out common numerical factors</td>
<td>( x'^2 - 10x + 19 = 0 )</td>
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| c. Matching to the standard form pattern, substitute \( a, b, \) and \( c \) into the quadratic equation. \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) | \( a = 1 \)
| c. = 19 | \( b = -10 \)
| c. = 19 | \( c = 19 \)
| d. Simplify | \( x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(19)}}{2(1)} \) |
|   | \( x = \frac{10 \pm \sqrt{100 - 76}}{2} = \frac{10 \pm \sqrt{24}}{2} = \frac{5 \pm 2\sqrt{6}}{2} = 5 \pm \sqrt{6} \) |

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And do not quit when the solution gives you an imaginary answer.

\[ i^2 = -1 \]

So \( \sqrt{-20} = \sqrt{-4 \cdot 5} = 2i\sqrt{5} \)