Ellipses (6.2), Circles (6.2) and Hyperbolas (6.3)

**Circle:** All the points equidistant from the center. That distance is the radius.

**Ellipse:** All the points such that the sum of the distance to two fixed points (foci) is constant.

**Hyperbola:** All the points such that the difference of the distance to two fixed points (foci) is constant.

**Circle:** really just a special case of an ellipse, where the 'two' foci are just one point

**Circle:**  
Standard form \((x-h)^2 + (y-k)^2 = r^2\) where center is \((h,k)\) and radius is \(r\)
General form (sometimes called expanded form) \(ax^2 + ay^2 + bx + cy + d = 0\)

The general form can be transformed into the standard form by completing the square.

Example 1: Graph and give the center and radius for \((x-3)^2 + (y-5)^2 = 4\)
Example 2: Graph and give the center and radius for \((x+2)^2 + y^2 = 9\)
Example 3: Graph and give the center and radius for \(x^2 + y^2 - 10x + 12y + 52 = 0\)
Example 4: Graph and give the center and radius for \(5x^2 + 5y^2 + 50x + 80y = 0\)

**Ellipse centered at origin:** \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) with two foci a distance of \(c = \sqrt{a^2 - b^2}\) from the center

Foci and Vertices on longer axis 
**Intercepts** are \((a,0), (-a,0), (0,b),(0,-b)\)

Example 5: Graph, labeling all the intercepts for \(25x^2 + 4y^2 = 100\)
Example 6: Graph, labeling all the intercepts for \(4x^2 + 25y^2 = 100\)
Example 7: Graph, labeling all the intercepts for \(16x^2 + y^2 = 16\)
Example 8: Graph, labeling all the intercepts for \(x^2 + 5y^2 = 5\)

**Hyperbolas centered on origin:**

\(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\) \quad \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1\)

Two foci a distance of \(c = \sqrt{a^2 + b^2}\) from the center
Foci and **Vertices** on axis of positive term’s variable when in standard form above.
No intercept on the axis of the negative term’s variable when in standard form above.

Asymptotes: \( y = \frac{\text{square root of } y \text{ denom.}}{\text{square root of } x \text{ denom.}} \), \( y = -\frac{\text{square root of } y \text{ denom.}}{\text{square root of } x \text{ denom.}} \).

**Example 9:** Graph, labeling all the intercepts for \( 4x^2 - 36y^2 = 36 \)
**Example 10:** Graph, labeling all the intercepts for \( 25x^2 - 9y^2 = 225 \)
**Example 11:** Graph, labeling all the intercepts for \( 36y^2 - 4x^2 = 36 \)

---

**71. Bridge Arch.** A bridge with a semielliptical arch spans a river as shown below. What is the clearance 6 ft from the riverbank?

---

**40. Nuclear Cooling Tower.** A cross section of a nuclear cooling tower is a hyperbola with equation
\[
x^2 - \frac{y^2}{90^2 - 130^2} = 1.
\]

The tower is 450 ft tall and the distance from the top of the tower to the center of the hyperbola is half the distance from the base of the tower to the center of the hyperbola. Find the diameter of the top and the base of the tower.