Useful Formulas

\[
\log_a x = y \iff a^y = x \quad \log_a M = \log_a N \iff M = N
\]

\[
\log_a MN = \log_a M + \log_a N \quad \log_a \frac{M}{N} = \log_a M - \log_a N \quad \log_a x = \frac{\ln x}{\ln a}
\]

\[
\log_{a^r} M = r \log_a M \quad a^{\log_a M} = M \quad \log_a a^M = M
\]

A = A_0e^{rt} \quad P = \frac{c}{a + ae^{-rt}} \quad u = T + (u_0 - T)e^{kt} \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}

1. Rewrite in logarithmic form

   \( e^b = d \) \quad \ln d = h \quad \text{or} \quad \log_e d = h

   \( 2^x = f - 3 \) \quad \log_2 (f - 3) = x - 5

2. Rewrite in exponential form

   \log (3x) = k \quad \ln (x - 5) = 2c \quad \log_3 (5x) = 12

3. Write the expression as the sum or difference of logs. Express powers as factors.

   \[
   \ln \frac{(x + 3)^3}{2x - 5} = 3 \ln (x + 3) - \ln (2x - 5)
   \]

4. Write the expression as a single logarithm.

   \[2 \log (x + 4) - \frac{1}{2} \log (x - 7) = \log \frac{(x + 4)^2}{\sqrt{x - 7}}\]

5. Use the Change-of-Base formula and a calculator to approximate the following to 3 decimal places.

   \[
   \log_a 3 = \frac{\ln 3}{\ln 6} \quad \text{or} \quad \log_6 3 = \frac{\ln 3}{\ln 6}
   \]

6. Solve the equation. Give both the exact and the decimal approximation to 3 places where appropriate.

   a. \( \ln(x - 2) = 3 \quad e^3 = x - 2 \)

   \[e^3 + 2 \approx 22.086\]
b. \[ 3e^{x+3} = 18 \]
\[ \ln(x) = x + 3 \]
\[ \ln(x) - 3 \approx 1.208 \]
\[ x = 1, 208 \]

c. \[ \frac{x^2}{36} = \log \left( \frac{x^2}{36} \right) = 2 \]
\[ 36 \cdot 10^2 = \frac{x^2}{36} \cdot 36 \]
\[ 36 \cdot 100 = x^2 \]
\[ 3600 = x^2 \]
\[ x = 60 \quad \text{or} \quad x = -60 \]

d. \[ \log_3(x) + \log_3(x+8) = \log_3(2x) \]
\[ \log_3(x^2 + 8x) = \log_3(2x) \]
\[ x^2 + 8x = 2x \]
\[ x^2 + 6x = 0 \]
\[ x(x+6) = 0 \]
\[ x = 0 \quad \text{or} \quad x + 6 = 0 \]
\[ x = -6 \]

\[ \text{No real solution} \]

\[ 4^{x-2} = 3^{2x+1} \]
\[ \ln(4^{x-2}) = \ln(3^{2x+1}) \]
\[ (x-2) \ln(4) = (2x+1) \ln(3) \]
\[ x \ln(4) - 2 \ln(4) = 2x \ln(3) + \ln(3) \]
\[ x \ln(4) - 2x \ln(3) = \ln(3) + 2 \ln(4) \]
\[ \frac{\ln(3) + 2 \ln(4)}{\ln(4) - 2 \ln(3)} \approx -4.774 \]
7. The number of woodpeckers in the conservation area was 300 in 1990. In 2000 it was 500. If the population exponential growth, find r, the growth rate during that time.

\[ A = A_0 e^{rt} \]

\[ \frac{500}{300} = e^{10r} \]

\[ \ln \left( \frac{500}{300} \right) = 10r \]

\[ r = \frac{\ln \left( \frac{5}{3} \right)}{10} = 0.051082 \ldots \]

\[ 0.0511 \]

round to four places

8. Use the above equation to predict the population in the year 2005.

\[ A_0 = 300 \quad t = 15 \quad r = 0.0511 \]

\[ A = 300 e^{0.0511 \times 15} \]

\[ 646 \]

9. A pan of brownies is cooling on the counter in the 75° kitchen. The brownies were removed from the 350° oven at 6 pm. At 6:10 pm their temperature was 280°. If their behavior followed Newton’s Law of Cooling, when will they be a pleasantly warm 115°?

\[ u = T + (u_0 - T) e^{-kt} \]

**First find k**

\[ u_0 = 350 \quad T = 75 \quad u = 280 \quad t = 10 \]

\[ 280 = 75 + (350 - 75) e^{10k} \]

\[ 205 = 275 e^{10k} \]

\[ \ln \left( \frac{205}{275} \right) = 10k \]

\[ k = -0.029381119 \ldots \]

**Now find t for when u = 115**

\[ 115 = 75 + (350 - 75) e^{-0.029381119t} \]

\[ 40 = 275 e^{-0.029381119t} \]

\[ 40 = e^{-0.029381119t} \]

\[ \ln (40) = -0.029381119t \]

\[ t = \frac{\ln (40)}{-0.029381119} = 266 \]

6 pm plus 66 minutes

\[ 7:06 \text{ pm} \]

10. Te-99m is routinely used for bone scans. If you start with a 10 gram sample of Te-99m, you will have 1 gram in 20 hours. What is the half life?

**Find r**

\[ A_0 = 10 \quad A = 1 \quad t = 20 \]

\[ 1 = 10 e^{20r} \]

\[ 0.1 = e^{20r} \]

\[ \ln 0.1 = 20r \]

\[ r = \frac{\ln 0.1}{20} \approx -0.115 \]

**Find the life for example A_0 = 2 A = 1**

\[ 1 = 2 e^{-0.115t} \]

\[ 0.5 = e^{-0.115t} \]

\[ \ln 0.5 = -0.115t \]

\[ t = \frac{\ln 0.5}{-0.115} \approx 6.02 \text{ hours} \]