Name Key Oct 2007 Review B MA 133

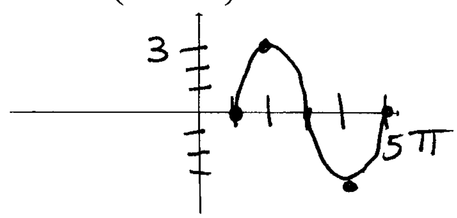
Write your answer, including units, on the line provided.
Show work on this paper or lose credit.

NOT ACCEPTED LATE Due October 25

1. For each of the following functions, give the amplitude, period, phase shift, vertical shift, and graph over the indicated interval.

a. $y = 3\sin\left(\frac{1}{2}x - \frac{\pi}{2}\right)$, Graph one complete cycle

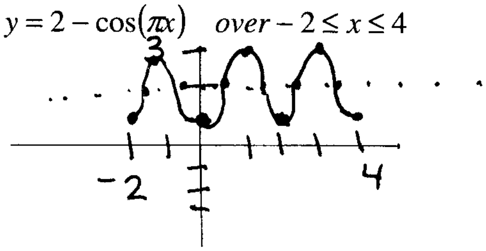
$p = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$



$PS = -\frac{c}{B} = \frac{-\left(-\frac{\pi}{2}\right)}{\frac{1}{2}} = \frac{\pi}{2} \cdot \frac{2}{1} = \pi$

A = 3 p = 4π PS = π VS = 0

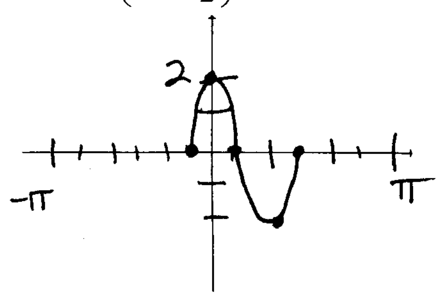
b. $y = 2 - \cos(\pi x)$ $over -2 \le x \le 4$



$p = \frac{2\pi}{\pi} = 2$

A = 1 p = 2 PS = 0 VS = 2

c. $y = 2\sin\left(3x + \frac{\pi}{2}\right)$, Graph one complete cycle

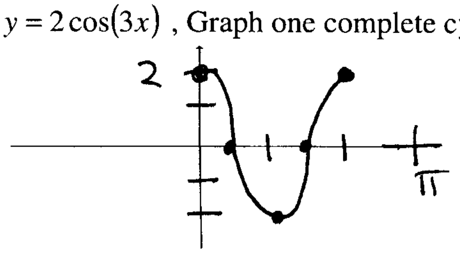


$p = \frac{2\pi}{B} = \frac{2\pi}{3}$

$PS = -\frac{c}{B} = \frac{-\frac{\pi}{2}}{3} = -\frac{\pi}{2} \cdot \frac{1}{3} = -\frac{\pi}{6}$

A = 2 p = $\frac{2\pi}{3}$ PS = $-\frac{\pi}{6}$ VS = 0

d. $y = 2\cos(3x)$, Graph one complete cycle



$p = \frac{2\pi}{B} = \frac{2\pi}{3}$

A = 2 p = $\frac{2\pi}{3}$ PS = 0 VS = 0

Remember: Show work
Plot 5 points per cycle.

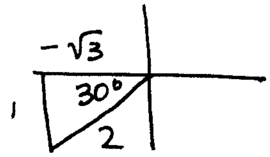
Alternate method

2. Simplify each of the following expressions: Write your answer in exact radians.

$$\begin{array}{r}180\\30\\\hline 210\end{array}$$

a. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, angle in QIII

$210^\circ\left(\frac{\pi}{180^\circ}\right)$

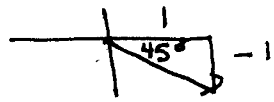


$\frac{7\pi}{6}$ $\pi + \frac{\pi}{6}$

$$\begin{array}{r}360\\-45\\\hline 315\end{array}$$

b. $\cot^{-1}(-1)$, angle in QIV

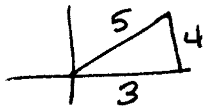
$315^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{4}$



$\frac{7\pi}{4}$ $2\pi - \frac{\pi}{4}$

3. Simplify each of the following expressions with EXACT answers. Assume angles in Q

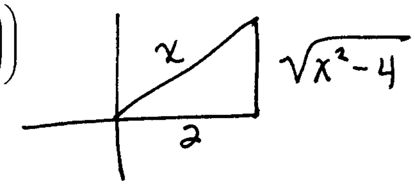
a. $\cos\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$



$\frac{3}{5}$

$2^2 + m^2 = x^2$
$4 + m^2 = x^2$
$m^2 = x^2 - 4$

b. $\sin\left(\cos^{-1}\left(\frac{2}{x}\right)\right)$



$\frac{\sqrt{x^2-4}}{x}$

4. Prove each of the following identities. You may use any of the formulas provided.

a. $(1 - \cos A)(1 + \cos A) = \sin^2 A$

$(1-\cos A)(1+\cos A) = 1 - \cos^2 A = \sin^2 A$

b. $\frac{\cot A}{\cos A} = \csc A$

$\frac{\cot A}{\cos A} = \frac{\frac{\cos A}{\sin A}}{\cos A} = \frac{\cos A}{\sin A} \cdot \frac{1}{\cos A} = \frac{1}{\sin A}$

$= \csc A$

c. $\sin^2 A = \frac{1 - \cos(2A)}{2}$

$\frac{1 - \cos(2A)}{2} = \frac{1 - [1 - 2\sin^2 A]}{2}$

$= \frac{1 - 1 + 2\sin^2 A}{2} = \frac{2\sin^2 A}{2} = \sin^2 A$

Remember: Start AND end with a given side.

d. $\tan x(\cos x + \cot x) = \sin x + 1$

$$\tan x(\cos x + \cot x) = \frac{\sin x}{\cos x}\left(\cos x + \frac{\cos x}{\sin x}\right)$$

$$= \frac{\sin x}{\cos x}\cdot \cos x + \frac{\sin x}{\cos x}\cdot\frac{\cos x}{\sin x} = \sin x + 1$$

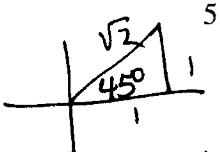
e. $\cos^2\left(\frac{A}{2}\right) = \frac{\tan A + \sin A}{2\tan A}$

$$\cos^2\frac{A}{2} = \left(\pm\sqrt{\frac{1+\cos A}{2}}\right)^2 = \frac{1+\cos A}{2}\cdot\frac{\tan A}{\tan A} = \frac{\tan A + \cos A\tan A}{2\tan A}$$

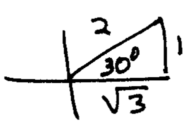
$$= \frac{\tan A + \cos A\left(\frac{\sin A}{\cos A}\right)}{2\tan A} = \frac{\tan A + \sin A}{2\tan A}$$

Note: Do not write cos tan instead of the proper cos A tan A.

5. Use the standard formulas to find **exact** values for the following Give your answer as a simplified **fraction**. Radicals in the denominator are acceptable. Other methods may be used.



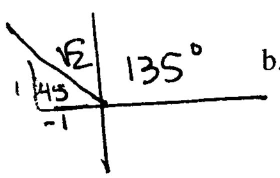
a. $\sin 15° = \sin(45° - 30°)$



$$= \sin 45° \cos 30° - \sin 30° \cos 45°$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\frac{\sqrt{3}-1}{2\sqrt{2}}$$



b. $\cos 165° = \cos(135° + 30°)$

$$= \cos 135° \cos 30° - \sin 135° \sin 30°$$

$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$\frac{-\sqrt{3}-1}{2\sqrt{2}}$$

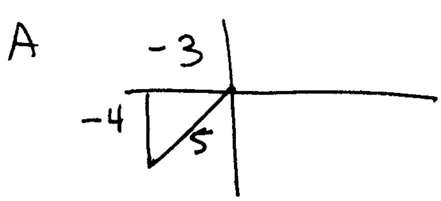
6. Let sin A = -4/5 with 180 ≤ A ≤ 270° and cos B = 5/13 with 270 ≤ B ≤ 360° to find EXACT values for the following. Give your answer as a simplified fraction.

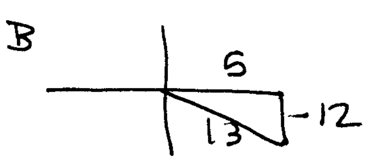
a. $\sin(A - B) = \sin A \cos B - \sin B \cos A$

$$= \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right)$$

$$= -\frac{20}{65} + \frac{36}{65}$$

$$-\frac{56}{65}$$





b. $\cos 2B = \cos^2 B - \sin^2 B = \left(\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2$

$= \frac{25}{169} - \frac{144}{169}$ $\qquad$ $\underline{-\frac{119}{169}}$

$180 < A < 270°$

$90 < A < 135$

c. $\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\sqrt{\frac{1+(-3/5)}{2}}$

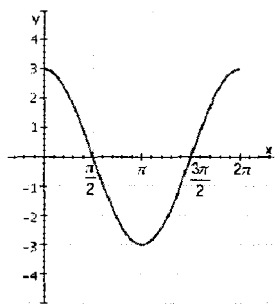
$= -\sqrt{\frac{\frac{2}{5}}{2}} = -\sqrt{\frac{2}{5}\cdot\frac{1}{2}} = -\sqrt{\frac{1}{5}}$ $\qquad$ $\underline{-\frac{1}{\sqrt{5}}}$

d. $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)$

$= -\frac{15}{65} + \frac{48}{65}$ $\qquad$ $\underline{\frac{33}{65}}$

7. Give an equation to fit the graph. Give the amplitude, period, phase shift and vertical shift:

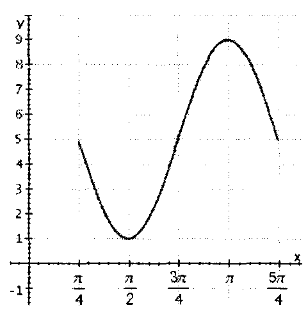


$p = \frac{2\pi}{B} = 2\pi$ $\qquad$ $B = 1$

a. A = $\underline{3}$ p = $\underline{2\pi}$ PS = $\underline{0}$ VS = $\underline{0}$

Eqn: $\underline{y = 3\cos x}$

DON'T FORGET $y=$



$p = \frac{2\pi}{B} = \pi$ $\qquad$ $B\pi = 2\pi$ $\qquad$ $B = 2$

$PS = -\frac{C}{B}$ $\qquad$ $\frac{-C}{2} = \frac{\pi}{4}$ $\qquad$ $-4C = 2\pi$ $\qquad$ $C = \frac{2\pi}{-4}$

b. A = $\underline{4}$ p = $\underline{\pi}$ PS = $\underline{\frac{\pi}{4}}$ VS = $\underline{5}$

Eqn: $\underline{y = -4\sin\left(2x - \frac{\pi}{2}\right) + 5}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin(A + B) = \sin A \cos B + \sin B \cos A$

$\sin(A - B) = \sin A \cos B - \sin B \cos A$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$