MA 134 ***Common Final Problem Set for 2009-2010***

Label word problem answers correctly. Show all work for full or partial credit. Give exact, simplified answers unless otherwise specified (e.g., $\frac{3}{2}$ instead of $\frac{15}{10}$, and $3\sqrt{2}$ instead of $\sqrt{18}$).

1. Find all solutions for
   a) $x^2 + 6x + 12 = 0$ c) $0 = 2x^2 - 2x + 5$ e) $x^3 + 2x^2 - 3x > 0$
   b) $x^2 - 4x = 8$ d) $x^3 + x^2 - 11x - 3 = 0$ f) $\frac{(x-3)(x+2)}{x-1} \le 0$

2. Give an equation of the circle with a radius of $\sqrt{5}$ and center at (3, -2).

3. Give the center and radius for the circle with the given equation:
   a) $x^2 + y^2 - 6x + 10y - 1 = 0$
   b) $2x^2 + 2y^2 - 12x + 8y - 24 = 0$
   c) $(x-3)^2 + (y+2)^2 = 36$

4. Graph each of the following. Sketch the original graph, and make a separate sketch for each stage of the transformation. Label at least two points on the original graph, and label the transformation of those points in each stage. Sketch and label any asymptotes.
   a) $g(x) = -\frac{1}{3}x^3 + 4$ b) $y = -3^x + 1$ c) $h(x) = -\sqrt{x+300} + 500$
   d) $y = \log_2(x+4) + 3$ e) $y = -e^{x-4} - 2$

5. Find the equation of the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$:
   Reflect about the y-axis, then reflect about the x-axis, then shift up 2 units.

6. Find the equation of the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$:
   Reflect about the x-axis, then shift left 100 units, then shift up 200 units.

7. For g(x) = -f(x - 3) + 5 , explain how the graph of g(x) has been transformed from the graph of f(x).

8. For g(x) = 2f(x + 7) – 14, explain how the graph of g(x) has been transformed from the graph of f(x).

9. Find the domain of
   a) $f(x) = \frac{x-5}{x^2 - 7x - 18}$ b) $f(x) = \sqrt{3-8x} + 5x$ c) $f(x) = e^{x-3}$ d) $f(x) = x^2 - 4x$
   e) $y = \ln(7-x)$ f) $f(x) = \frac{6}{\sqrt{x+8}}$

10. Given the functions $f(x) = \sqrt{x-2}$ and $g(x) = x^2 - 1$, find the following:
    a) $f(3) + g(3)$ b) $(g \cdot f)(x)$ c) $(g \circ f)(x)$ d) $f(a)$
    e) $f(x) + 3$ f) $f(x+3)$ g) $g(-x)$ h) $-g(x)$
    i) $g(x+h)$ j) $(f/g)(x)$ and state the domain k) $f(g(x))$

11. Find $\frac{f(x+h) - f(x)}{h}$ for a) $f(x) = 3x - 5$ b) $f(x) = x^2 + x - 5$

12. Determine if $g(x) = 3x^2 - 6x + 1$ has a maximum or minimum value, and find the maximum or minimum value.

13. Graph, accurately plotting x-intercepts and any horizontal or vertical asymptotes, and give the location of any holes in the graph. Label all the asymptotes.

a) $y = (x-4)^3(x+3)^2$ b) $f(x) = \dfrac{(x-5)^2}{x^3 - 9x}$ c) $y = 2x^2(x+4)^2(x-3)^3$

d) $h(x) = \dfrac{x^2}{x^2+2x-15}$ e) $7x^2 + 4y^2 = 28$ f) $f(x) = \dfrac{4}{x^2+1}$

g) $y = \dfrac{x^2+4x-21}{x^2-7x+12}$ h) $f(x) = x^3 - x$ i) $100x^2 + 25y^2 = 100$

j) $g(x) = \dfrac{x-1}{x-2}$ k) $y = 5x^2 + 10x + 5$ l) $y^2 - 4x^2 = 16$

m) $h(x) = \begin{cases} x+3 & \text{if } x<0 \\ 4-x & \text{if } x \ge 0 \end{cases}$ n) $f(x) = \begin{cases} 2+x & \text{if } x<1 \\ 5 & \text{if } x=1 \\ x^2 & \text{if } x>1 \end{cases}$ o) $y^2 = -12x$

14. Consider the equation $y = -2(x+a)^3(x-b)^2$, where *a* and *b* are positive real numbers.
   a) Find the *y*-intercept(s)
   b) Find the *x*- intercepts and indicate whether the graph crosses or touches at each.
   c) Give the end behavior as *x* approaches positive infinity -- i.e., as $x \to \infty, y \to$ ___________
   d) Give the end behavior as *x* approaches negative infinity -- i.e., as $x \to -\infty, y \to$___________

15. Write $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$ as a product of linear factors.

16. Find all zeros of $f(x) = x^3 + 13x^2 + 57x + 85$

17. Find the inverse of each of the following. Give the domain and range of the function and its inverse. a) $g(x) = 4 - 3x$
   b) $f(x) = \dfrac{6}{x+1}$ c) $y = x^5 - 2$ d) $f(x) = \dfrac{2x}{x-3}$ e) $y = \sqrt[3]{x+5}$

18. Write $y = 4^{3x+5}$ as a logarithmic equation.

19. Write $m = \log_2(p+3)$ as an exponential equation.

20. a) Write $3\ln(x+2) + \frac{1}{2}\ln x - \ln(x-1)$ as a single logarithm.
   b) Write $\log_4\left(x^3\sqrt{x-5}\right)$ as a sum or difference of logarithms. Express powers as factors.

21. Solve each of the following equations. Give both exact answers and approximate answers to 3 decimal places (where appropriate).

a) $\log_3(x-4) + 2 = 7$ d) $\log_4 x + \log_4(x-3) = 1$ g) $0 = \log_2(x+4) + 3$

b) $200 = 500e^{-0.24d} + 100$ e) $2^{3x-5} = 6^x$ h) $2 = e^{x-4} - 7$

c) $\log_5(2x+3) = 2\log_5 3$ f) $0 = 3^x - 8$ i) $\log_3(4x+2) - \log_3(x) = 2$

22. An object is thrown vertically up and its height after *t* seconds is given by the formula $h(t) = 96t - 16t^2$.
   a) Find the maximum height attained by the object.
   b) After how many seconds does it attain the maximum height?
   c) After how many seconds does it return to its starting position?

23. Give both exact answers and approximate answers to 3 decimal places, where appropriate.
    a) Suppose the half-life of Girardium is 20 minutes. Find how much will be left in 105 minutes if you start with 100 g.
    b) A culture of bacteria obeys the law of exponential growth. If 500 bacteria are present initially, and there are 800 after 1 hour, how many will be present in the culture after 5 hours?

24. The number of watts *w* provided by a space satellite's power supply after *d* days is given by $w(d) = 50e^{-0.004d}$. a) How much power is provided after 30 days? Give both the exact answer and an approximate answers to 3 decimal places, where appropriate.
    b) How many days will elapse before there are 41 watts left? Give both the exact answer and an approximate answer to the nearest day.

25. Joe has available 200 meters of fencing and wishes to enclose a rectangular field along a bluff. He does not need to fence the edge along the bluff. What are the dimensions of the field with the largest possible area?

26. You have found that price (p) in dollars is related to the number of tickets sold (x) by the equation $p = -2x + 48$. Using the fact that revenue is the product of price times the number sold, find the ideal price for your tickets to maximize revenue. Give both the ideal price and the number of tickets sold.

27. The average weight of a baby born in 1900 was 6.25 pounds. In 2000, the average weight of a newborn was 6.625 pounds. We will assume for our purposes that the relationship is linear. Find an equation that relates the year to the average weight of a newborn. Using that equation, predict the average weight of a newborn in 2035.

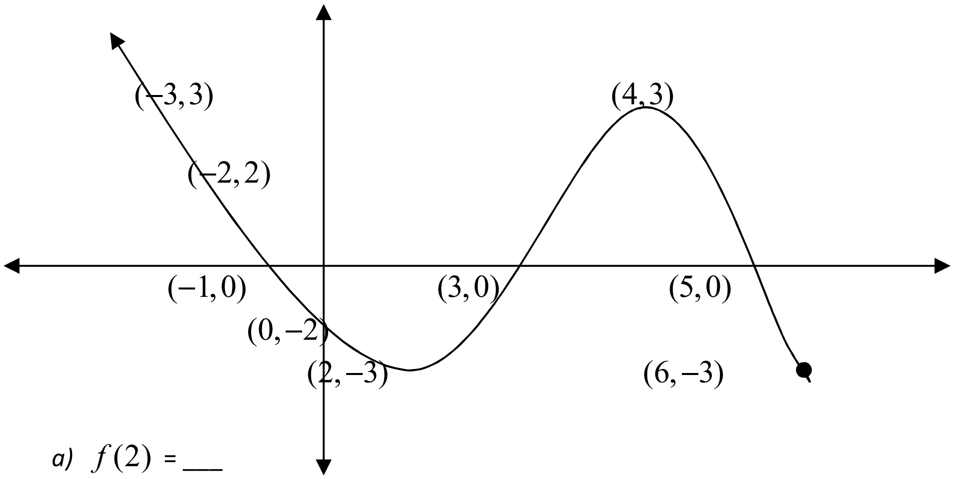
28. The relationship between Celsius (°C) and Fahrenheit (°F) is linear. We know that 0°C corresponds to 32°F, and that 100°C corresponds to 212°F. *Derive* an equation that relates °C to °F. Use that equation to find the Celsius equivalent of 37°F.

29. Find the first 5 terms for each: a) $a_n = (-2)^{n+1}(n+4)$ b) $a_n = 3 - 2a_{n-1}; a_1 = 4$

30. Each of the following is arithmetic or geometric.
    a) Find the 100th term of 3, 7, 11, 15, 19,…
    b) Find the nth term of 10, 6, 2, -2, -6, …
    c) Find the 100th term and the sum of the first 100 terms of 7, 10, 13, 16 . . .
    d) Find the sum of the first 500 natural numbers: 1 + 2 + 3 + 4 + ….
    e) Find the 40th term of 2, 6, 18, 54, …
    f) Evaluate $\sum_{k=0}^{100}(4k+3)$.
    g) Evaluate $\sum_{k=3}^{6}k(k+2)$.
    h) Find the sum of the first 50 terms of $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\ldots$
    i) Find the sum: $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\ldots$
    j) Evaluate $\sum_{k=1}^{\infty}6(0.4)^k$.

31. Construct a rational function which has vertical asymptotes $x = 1$ and $x = -4$, horizontal asymptote $y = -5$ and x-intercepts 2 and 3.

32. Use the graph of the function $f(x)$ to answer questions *a – h*:



a) $f(2)$ = ___

b). For what value(s) of *x* is $f(x)=3$?

c) The domain of *f* is ___

d) The range of *f* is ___

e) For what interval(s) is $f(x)$ decreasing?

f) For what interval(s) is $f(x)$ increasing?

g) There is a relative maximum of ____________ at ______________.

h) There is a relative minimum of ____________ at ______________.

i) Use the graph to solve $f(x) < 0$.

j) Find the real zeroes of the function.

k) $f(0)$ = _____

Disclaimer: There are more than 100 problems on this Problem Set, but there will be only about 20-25 problems on the Final. In general, the questions on the Final will be like these questions, but there may be minor variations. Anyone who understands all of these problems should do well on the Final.

Problem Set problems keyed to chapters in Beecher

Chapter	Problem Set Problems
1	5, 6, 7, 8, 9, 10, 11, 27, 28, 32
2	11, 22, 25, 26
3	14, 15, 16, 31
4	4, 9, 17, 18, 19, 20, 21, 23, 24
6	13
7	29, 30

Suggestion: After finishing Chapter 1, do the Chapter 1 Problem Set Problems; and after finishing Chapter 2, do the Chapter 2 Problem Set Problems, and so on.