Calculus III
Revision Exercises Chapter 14.2-14.8; 15.1

1. Find the limit, if it exists, or show that the limit does not exist. (a) $\lim_{(x,y)\to(-1,1)} \ln(1+y)\sin\left(x^2+y^2\right)$, (b) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$, (c) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2-y^2}$.

2. Find the first partial derivatives of the functions (a) $f(x,y)=\sqrt{x^3}\ln y$, (b) $f(x,y)=\sec\left(2xy^2\right)$, (c) $f(x,y,z)=xye^{2x/z}$.

3. If (a) $f(x,y)=x\sqrt{\sin x\cos y}$, find $f_x(\pi/4,\pi/6)$, (b) $f(x,y)=\tan^{-1}(x/y)$, find $f_x(4,1)$.

4. Find all the second partial derivatives a. $w=xe^{xy^2}$, b. $w=\dfrac{y^3}{x^2+y^2}$.

5. Find the equation of the tangent plane of (a) $z=x^2y^{-1/2}+y^{-3}$ at $(2,1,5)$, (b) $z=e^{x^2+y^2}$ at $(0,1,e)$.

6. Find the linear approximation of the function $f(x,y,z)=\frac{x^2}{y^3+z}$ at $(1,2,1)$ and use it to approximate $\frac{0.98^2}{2.01^3+1.05}$.

7. The volume of a cylinder of radius r and height h is $V=\pi r^2h$. (a) Show that $dV=\pi 2rhdr+\pi r^2dh$. (b) Calculate the approximate increase in V (using dV/V) if r and h are each increase by 2%.

8. Evaluate dw/dt at the given value of t when (a) $w=\dfrac{x^2}{z}-\dfrac{y^2}{z}$, $x=\sin t$, $y=\cos t$, $z=\dfrac{1}{t}$; (b) $w=z\ln\left(x+y^2\right)$, $x=\sqrt{t}$, $y=e^{-2t}$, $z=\ln t$.

9. Find $\partial w/\partial r$ and $\partial w/\partial s$ when $w=\tan\left(x^2-y\right)$, $x=r+4s$, $y=2r-5s$.

10. Find $\partial w/\partial u$ and $\partial w/\partial v$ at the given point $w=\sqrt{1+x^2+y^2+z^2}$, $x=uv$, $y=u+4v$, $z=5u-v$; at $(u,v)=(1,-1)$.

11. Find dy/dx when (a) $x^2-3xy+y^4=5$; $(b)\, x\cos y-y\sin x=e^{2x}$.

12. Find the directional derivative of the function at P_0 in the direction of A: (a) $f(x,y,z)=x^3y-4xy^3$, $P_0(1,-2)$, $A=2i-j$; (b) $f(x,y,z)=\sqrt{x-y}\sin(y+z)$, $P_0(2,-1,0)$, $A=i+2j+2k$.

13. Find the maximum rate of change of f at the given point and the direction in which it occurs when (a) $f(x,y)=\tan(xy)$, $P_0(\pi/4,1)$; (b) $f(x,y,z)=xe^{2z}+ye^{-3x}-ze^y$, $P_0(1,-1,0)$.

14. A bug located at (3, 9, 4) begins walking in a straight line toward $(5,7,3)$. At what rate is the bug's temperature changing if the temperature is $T=xe^{y-z}$?

15. Find the local maximum minimum values and saddle points of the following functions when (a) $f(x,y)=2x^2+y^2+2xy+2x+2y$; (b) $f(x,y)=e^y\left(y^2-x^2\right)$.

16. Find the absolute maxima and minima of $f(x,y)=x^2+xy+y^2-3x+3y$ in D which is (a) the closed triangular region with vertices $(0,0)$, $(4,0)$, and $(0,4)$, and (b) the closed rectangular region with $0\le x\le 2$, $0\le y\le 3$.

17. Find the minimum and maximum values of the function subject to the given constraint when (a) $f(x,y)=x^2y^4$, $x^2+2y^2=6$; (b) $f(x,y,z)=xy+3xz+2yz$, $5x+9y+z=10$.

18. Use the midpoint rule to estimate $\int_0^2\int_0^3\left(x^2+6y^2\right)dA$ with $m=n=2$.